The height of origin of muons in large air showers

1973 J. Phys. A: Math. Nucl. Gen. 6 1244

(http://iopscience.iop.org/0301-0015/6/8/018)

J. Phys. A: Math., Nucl. Gen., Vol. 6, August 1973. Printed in Great Britain.

The height of origin of muons in large air showers

J C Earnshaw†, A C Machin‡, D R Pickersgill§ and K E Turver
Department of Physics, University of Durham, South Road, Durham, UK

Received 15 January 1973, in final form 21 March 1973

Abstract. The mean heights of production of muons of various momenta in large air showers have been determined from measurements made with the Haverah Park magnet spectrograph of the lateral and angular scatter of muons from the shower core and the geomagnetic distortion of the muon charge ratio at various points in the shower front.

The heights of origin of muons of momentum p GeV/c recorded at distances of r km from the core are given by:

$$H(p, r) = H_0 + \alpha \lg p + r/\beta$$

where

$$H_0 = 1{\cdot}68 \pm 0{\cdot}15 \text{ km}$$

$$\alpha = 1{\cdot}74 \pm 0{\cdot}20 \text{ km}$$

$$\beta = 0{\cdot}263 \pm 0{\cdot}033.$$

1. Introduction

There is considerable contemporary interest in the longitudinal development of the extensive air showers (EAS) formed by primary cosmic rays having energies in excess of 10^{17} eV. This interest arises from the possibility that the longitudinal development of EAS may indicate the mass of the primary particle and from its relevance to the nuclear physics of the very high energy interactions.

Several experiments have been made in recent years to measure the distribution in spatial angles of EAS muons relative to one another, or to the EAS core direction, in attempts to determine the longitudinal shower development (de Beer *et al* 1962, Earnshaw *et al* 1968, de Beer *et al* 1970). In addition, the distortion of the charge ratio of muons recorded at prescribed lateral positions in EAS and arising from the interaction between the muons and the geomagnetic field has been suggested as a possible method of measuring the height of origin of muons (Somogyi 1966) and shown to be practicable by Orford *et al* (1968). Other workers (Khristiansen 1958, Clark *et al* 1958, Oren 1959, Kamiya *et al* 1962) have estimated the deflection of muons in the geomagnetic field, mainly in relation to the possible broadening effects on the muon lateral distribution function rather than as a means of determining their heights of origin.

To date these experiments have succeeded only in confirming the general properties of the origin of muons in the showers and their limited success has been due to either poor statistics, lack of precision of EAS and muon data, or an oversimplified analysis

† Now at the Queens University, Belfast.
‡ Now at Royal Aircraft Establishment, Farnborough.
§ Now at Rutland Sixth Form College, Oakham.

procedure. In view of the possible importance of the problem, further consideration of the possibilities outlined by Earnshaw (1968) based upon improved muon and EAS data and a refined analysis has been made and is the subject of this paper. The data from such measurements provide values of the observed mean height of production of muons at various momenta and core distances in large EAS independent of shower models and these values can be used as a test of model predictions for the distribution in height of origin of muons, appropriate allowance being made for the angular and geomagnetic effects in the propagation of muons through the atmosphere.

2. The experimental arrangement

The measurements of the momenta, electric charge and arrival direction of EAS muons have been made using the Haverah Park solid iron magnet spectrograph which has been fully described by Earnshaw *et al* (1967). The measurements have been made on muons of momentum 1–30 GeV/c recorded at core distances of 150–600 m in showers initiated by primary particles of energy about 10^{17} eV. The spectrograph was triggered and photographed whenever a shower was recorded by the large water Čerenkov detector EAS array at Haverah Park. The description of this array and its performance in recording and measuring large showers has been given by Tennent (1967).

3. The mean height of origin of muons in EAS from measurements of the arrival direction of muons relative to the shower core direction

3.1. *General description of the method*

The fundamental relation used in this method may be represented diagrammatically as in figure 1(a). A muon of momentum p arriving at a detector, a distance r from the core measured in the plane of the shower front, must originate from height H such that

$$\frac{p_t}{p} = \frac{r}{H} \tag{1}$$

where p_t is the transverse momentum possessed by the muon. This relation holds fairly well for momenta above 10 GeV/c at the core distances considered here; below this momentum the muon does not propagate linearly due to scattering effects, but the relation is still broadly correct. For example, a muon of momentum 2 GeV/c originating from a height of 2·5 km and being recorded at a core distance of about 300 m will have a (systematic) angular deviation arising from p_t which is 2–3 times as large as the (random) angular deviation arising from Coulomb scattering in the atmosphere. Equation (1) contains two unknowns, p_t and H; the form of the p_t distribution as suggested by Cocconi *et al* (1961)—the CKP distribution—leads to lateral distributions of muons in good agreement with experiment. Making the assumption that the CKP distribution is a valid representation of the p_t distribution and that its mean value is 0·4 GeV/c, a value of H may be derived by transforming equation (1) into a form more directly comparable with experimental measurements.

The main constraint on the present studies of the angular properties of muons is a result of the geometry of the spectrograph. The design of the instrument as a momentum analyser requires that the deflection of the muon be measured in one plane only; the

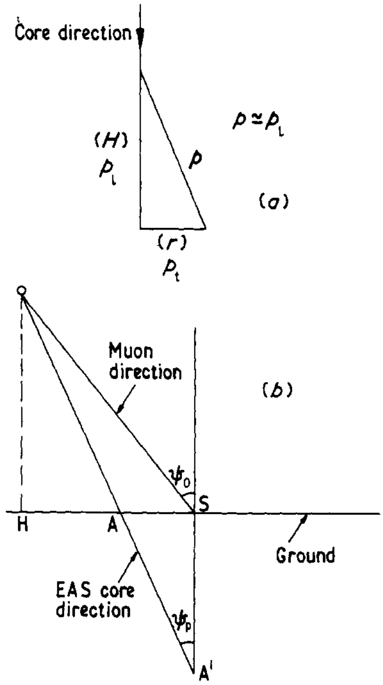

Figure 1. (*a*) The relation between muon momentum, core distance, height of origin and transverse momentum. (*b*) The relation used to determine the height of origin of a muon after projecting the shower arrival direction into the spectrograph measuring plane. The convention chosen is such that the angles shown would be negative.

angle of incidence of the muon on the spectrograph is also known in this plane and all measurements including the core arrival direction, must also be made in this plane. The angle made by the shower core with the vertical in the measuring plane of the spectrograph is ψ_p, given by:

$$\tan(\psi_p) = -\tan(\theta)\cos(\phi + E) \tag{2}$$

where θ, ϕ are the zenith and azimuth angles of the EAS respectively. The core distance measured along the ground from the spectrograph (also projected into the measuring plane) is required in the analysis and is found from the rotation of axes to be:

$$r_\perp = Y\cos E - X\sin E \tag{3}$$

where X, Y are the cartesian coordinates of the core impact point in the EAS array shown in figure 2. The angle E arising in equations (2) and (3) comes from the rotation of the spectrograph measurement plane through 34·5° with respect to the chosen array cartesian axes (see figure 2), and this rotation has some useful effects, as will be noted later.

Figure 1(*b*) shows the effect of projecting the core direction into the spectrograph measurement plane. OS is the projected direction of the muon, OAA' the projected direction of the core, and OH the height of origin required. Since AS is defined as $r_\perp$, the following relationships may be obtained:

$$H = r_\perp \frac{\cos(\psi_p)\cos(\psi_0)}{\sin(\psi_p - \psi_0)} \tag{4}$$

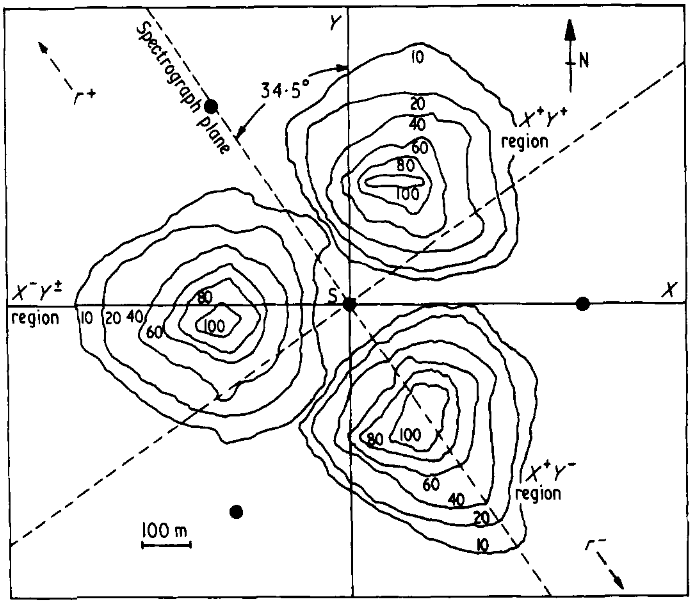

Figure 2. A sketch map of the Haverah Park 500 m EAS array showing the location of the large particle detectors (●), the magnet spectrograph (S) and the chosen cartesian coordinate system. Contours of equal numbers of showers containing measured muons per 100 m × 100 m bin of core impact coordinates X and Y are shown. The regions of the array X^+Y^+, X^+Y^-, $X^-Y^\pm$ containing the majority of the useful events are indicated.

or

$$\frac{r_\perp}{H} = \tan(\psi_p) - \tan(\psi_0). \tag{5}$$

Since the angle between the muon and the core direction $(\psi_p - \psi_0)$ is generally small (typically less than 10°), errors in the value of H arise mainly from errors in $(\psi_p - \psi_0)$. The form of equation (4) shows that it is more useful to work in terms of $1/H$, for the errors then relate more directly to those in $(\psi_p - \psi_0)$. Errors in $r_\perp$ are less important although care is necessary when using small values of $r_\perp$ and it has been customary to exclude showers falling such that $r_\perp$ is less than 10 m.

3.2. The effect of the collecting area of the large EAS *array on the recorded distribution in angular deviations of muons from the core*

3.2.1. The effect on $r_\perp$. The symmetrical layout of the 500 m EAS array and the triggering criterion (see eg Tennent 1967) causes the most frequent smaller showers to be detected preferentially when their cores fall away from the lines joining the outer stations to the centre of the array (see eg Suri 1966). The shower data comprising the present sample have been subdivided into intervals of core impact coordinates X and Y, and figure 2 shows the distribution of shower core impact points, in the form of contours of equal numbers of showers falling in 100 m × 100 m bins of X and Y. It has been noted above that the spectrograph plane is rotated through 34·5° with respect to the XY coordinates system, and it is apparent from figure 2 that more showers fall in the $r_\perp^+$ ($r_\perp$ positive) region than the $r_\perp^-$ ($r_\perp$ negative) region because of this effect, the ratio being about 1·5 : 1.

However, the value of $\langle r_\perp^- \rangle$ is larger than the value of $\langle r_\perp^+ \rangle$, an effect which will also be taken into account later.

3.2.2. The effect on the observed angle $(\psi_p - \psi_0)$. The heights of origin of all muons predicted from equation (4) would be positive in the absence of scattering. Inspection of this relation shows that if ψ_p and ψ_0 are of the same sign (usually so except for near-vertical showers), then $(\psi_p - \psi_0)$ must be the same sign as $r_\perp$ in order to give a positive value of H. Since $r_\perp$ may be divided into two populations, it is to be expected that such a division will produce two populations of $(\psi_p - \psi_0)$, one of predominantly positive values and one of predominantly negative values. Figure 3 shows the broadly negative tendency of the distribution in angle $(\psi_p - \psi_0)$ for $r_\perp^-$ events.

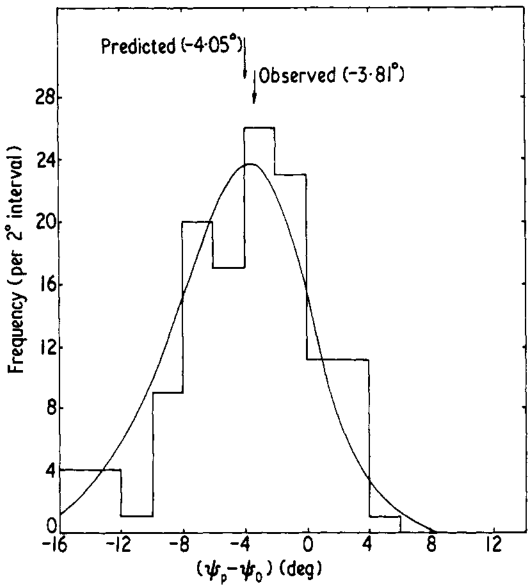

Figure 3. Distribution in the angle between the muon and the core distance for core distances between 250 and 350 m for 1–3 GeV/c muons. Also shown is the curve predicted by the shower model of Orford and Turver (1970).

3.3. Determination of the measurement error on the angle $(\psi_p - \psi_0)$

The overall uncertainty on measurements of shower zenith and azimuthal angles was estimated at about $\pm 2{\cdot}2°$ in θ and $\pm 7°$ in ϕ (see eg Hollows 1969). This, transformed into an error of the core direction in the spectrograph plane, ψ_p, yields an expected error of approximately $\pm 2{\cdot}5°$. The error on the muon arrival direction ψ_0 measured using the spectrograph is some $\pm 0{\cdot}3°$, so that the greatest contribution to the overall error in $(\psi_p - \psi_0)$ arises from the errors in θ and ϕ which come, in turn, from errors in fast timing. Application of equation (1) to muons of momentum greater than 30 GeV/c and the height of origin of 10 km detected at a typical core distance of 300 m shows that the angle $(\psi_p - \psi_0)$ arising from the transverse momentum of the parent pion is less than 1°. Thus the distribution in $(\psi_p - \psi_0)$ for such high momentum muons should give a good estimate of the overall uncertainty in the angular measurement. This distribution has been obtained using values ψ_p based on measurements of azimuth and zenith corrected for curvature of the shower front (see eg Suri 1966). There is good agreement between the RMS expected uncertainty in angle of $\pm 2{\cdot}5°$ and the observed value.

3.4. The average production height of muons by a direct method

The mean reciprocal height of origin has been determined for muons recorded at 16 intervals of momentum and core distance by direct application of equation (4). The ranges concerned are 1–3, 3–8, 8–15, 15–30 GeV/c for muon momentum and 150–250, 250–350, 350–450, 450–600 m for core distance. The results, found by calculating the mean value of $1/H$ and its standard deviation, then converting the values to heights of production, are shown in figure 4. Also shown are typical predictions from shower simulations when the primary particle is assumed to be a proton and an iron nucleus.

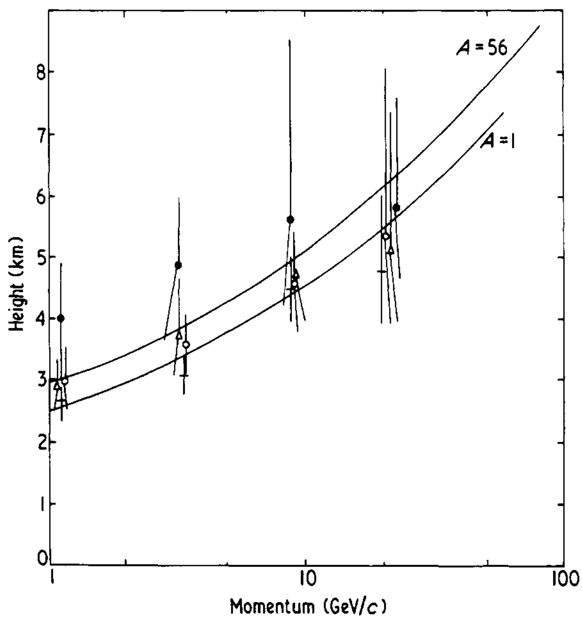

Figure 4. The height of production as a function of muon momentum for four bands of core distance. Model predictions for two primary masses are also shown for core distances of 200 m when the primary particle's energy was 2×10^{17} eV/nucleus. + 200 m; ○ 300 m; △ 380 m; ● 520 m.

3.5. An analytic representation of the height of production of muons of known momentum and core distance

This derivation was undertaken to obtain a simple relation and to allow direct comparison of the experimental results with those of other workers. Figure 5 shows the height of origin data plotted as a function of core distance for different muon momentum bands. The data were assumed to represent linear functions of core distance, so that the data of figure 5 may be represented by four parallel straight lines. Four least-squares fit lines were computed, and the mean and standard error calculated to find the slope. The four values for the constant term were then compared with the momentum dependence of the production height for a fixed core distance interval and resolved into a

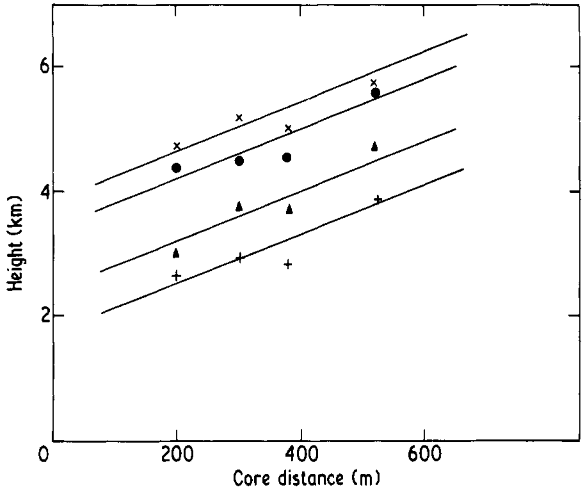

Figure 5. The mean height of production of muons as a function of core distance for the momentum bands shown. The four lines have the same gradient, equal to the mean of the least-squares fits to the four sets of data. + 1–3 GeV/c; △ 3–8 GeV/c; ○ 8–15 GeV/c; × 15–30 GeV/c.

constant term and a lg p term. A relation of the form:

$$H(p, r) = H_0 + \alpha \lg p + r/\beta$$

was obtained, where $H_0 = 1{\cdot}68 \pm 0{\cdot}15$ km, $\alpha = 1{\cdot}74 \pm 0{\cdot}20$ km, $\beta = 0{\cdot}263 \pm 0{\cdot}033$, and r is in kilometres. All the errors were determined from the spread in the least-squares calculations. The resulting equation represents the production height of muons fairly well for core distances of 150–600 m, and for momentum from 1–30 GeV/c.

4. The mean height of origin of muons from measurements of the distortion of the charge ratio of muons arising from the geomagnetic deflection of muons

One potential advantage of using the interaction between the muons and the geomagnetic field to investigate the heights of origin of muons is that (random) Coulomb scattering has very little effect on the (systematic) geomagnetic deflection of the muon, provided that the angular deflection is small, since the geomagnetic deflection is superimposed on all muons in the shower regardless of such scattering. Consequently, the deflection of muons in bulk in the geomagnetic field, can be thought of as being directly related to their height of origin. Measurement of this bulk deflection is possible by consideration of the distortion produced in the charge ratio of muons by the lateral shifts, in opposing directions, of positive and negative muons.

4.1. Calculation of the deflection of muons in the geomagnetic field and the resulting distortion of the charge ratio

4.1.1. Deflection in the geomagnetic field. The deflection of a positively charged muon in the earth's magnetic field has been evaluated, for given values of momentum and production height, for a muon which has an initial direction specified by zenith angle θ

and azimuth varying from 0° to 360°. The value of the horizontal component of the geomagnetic field strength appropriate to the location of Haverah Park was assumed to be 0·187 Oe, assumed constant for all altitudes of interest here. The value of declination was 9°W and that of inclination 68·5°. The value of muon momentum used in the calculations is a mean value of the initial and final momenta, after energy loss due to ionization has been taken into account. The mean deflection of a muon, weighted by the observed zenith angle distribution of showers detected by the Haverah Park EAS array, is obtained for a range of values of azimuth and the projected component of this mean deflection along the direction of each of the three lines passing through the areas of the array in which many shower cores impact (see figure 2) is then derived.

4.1.2. Distortion of the charge ratio by the geomagnetic field. The charge ratio of muons in EAS having cores which fall in each of the three areas of the array having a high triggering probability is obtained by assuming: (i) that in the absence of geomagnetic field the lateral distribution of positive muons is the same as that for negative muons; (ii) that when the field is considered to be present the lateral distributions, appropriate to μ^+ and μ^-, are separated by twice the mean geomagnetic deflection at that distance from the core; and (iii) that at a given distance the ratio of the ordinates of the lateral distributions appropriate to positive and negative muons gives a value of the expected charge ratio for muons. For the purposes of determining a mean height of origin a lateral distribution derived from the experimental data and appropriate to the mean momenta under consideration was used.

The estimation of the charge ratio for several distance intervals was not possible because the number of events was statistically insufficient, and therefore a single distance interval of 150 m to 600 m was considered. Figure 4 indicates that this procedure is valid. The distance at which the charge ratio was determined from calculations was the median distance for the data and was found to be 300 ± 10 m.

4.1.3. Typical predictions. Calculated values for the deflections of positive muons in the geomagnetic field appropriate to Haverah Park and originating at a vertical height H km above sea level and having a direction specified by zenith angle θ and azimuth angle ϕ, for two values of sea level momentum and two values of height of origin, are given in tables 1 and 2. Values for any other momentum/height/zenith/azimuthal combination may be obtained with sufficient accuracy by extrapolation or interpolation of these data.

4.2. Possible sources of inaccuracy in the method

It will be shown that the gross effect of the geomagnetic field on EAS muons at Haverah Park is detectable above any competitive process. Nevertheless, it is possible for the relationship between the distortion of the charge ratio of muons and their height of origin on the one hand, and the extent to which their relationship is sensitive to the shape of the lateral distribution function on the other, to be obscured by several sources of inaccuracy, both in the experimental method and in the theoretical calculations.

The uncertainty arising from the following sources of error has been considered and shown in all cases to be insignificant: (*a*) the muon charge ratio in an undistorted shower, assumed to be unity; (*b*) the variation of the geomagnetic field with altitude; (*c*) the effect of the earth's electric field on muon trajectories; (*d*) the effect of errors in shower

Table 1. The geomagnetic deflection in the horizontal geographic north and east directions of a positively charged muon of momentum 2 or 5 GeV/c at sea level in a shower having a core direction defined by zenith angle θ and azimuth angle ϕ

Sea level momentum (GeV/c)		2·0		2·0		5·0		5·0	
Height of origin (km)		2·5		15·0		5·0		10·0	
	ϕ (deg)	Deflection of muons (m)							
		East	North	East	North	East	North	East	North
$\theta = 0°$	0	5·7	0·0	187·8	0·0	11·2	0·0	42·5	0·0
$\theta = 10°$	0	9·8	0·0	274·6	5·5	16·4	0·1	62·3	0·4
	45	9·0	−2·0	253·1	−51·9	15·1	−3·3	57·4	−12·6
	90	7·0	−3·0	196·5	−80·5	11·7	−5·0	44·5	−18·9
	135	4·7	−2·2	133·5	−59·9	7·9	−3·7	30·2	−14·1
	180	3·7	0·0	104·8	2·4	6·3	0·0	23·8	0·2
	225	4·7	2·4	131·6	66·3	7·7	4·1	29·9	14·9
	270	6·9	3·3	194·5	89·6	11·4	5·6	44·1	20·1
	315	8·9	2·3	252·2	62·7	14·9	4·1	57·2	14·0
$\theta = 20°$	0	13·6	0·1	376·6	7·1	22·7	0·1	86·2	0·6
	45	12·1	−4·1	336·9	−107·8	20·3	−6·9	77·1	−25·8
	90	8·0	−6·5	223·5	−174·9	13·4	−10·8	50·9	−40·9
	135	2·9	−5·0	83·4	−138·2	4·9	−8·5	18·7	−32·0
	180	0·5	0·0	14·9	0·4	0·9	0·0	3·4	0·0
	225	2·7	5·2	78·0	143·5	4·5	8·7	18·0	32·6
	270	7·8	6·7	219·5	187·2	12·9	11·4	50·1	42·4
	315	12·0	4·5	336·1	122·6	20·0	7·7	76·7	27·6
$\theta = 30°$	0	18·6	0·1	509·1	9·2	31·3	0·1	118·4	0·7
	45	16·6	−6·5	453·9	−169·5	28·0	−10·9	105·7	−40·9
	90	10·1	−11·0	277·8	−293·1	17·0	−18·5	64·3	−69·6
	135	1·0	−9·1	32·3	−245·7	1·7	−15·2	6·7	−57·6
	180	−3·5	−0·0	−96·0	−2·8	−5·9	−0·0	−22·4	−0·2
	225	0·7	9·1	21·0	250·8	1·1	15·3	5·3	57·9
	270	9·8	11·3	272·0	312·5	16·3	19·1	63·1	71·6
	315	16·5	7·0	455·5	190·4	27·6	11·9	105·5	43·4
$\theta = 40°$	0	26·2	0·1	698·3	11·8	44·3	0·1	166·5	1·0
	45	23·9	−9·5	632·8	−242·5	40·4	−16·1	151·6	−60·1
	90	14·3	−17·7	382·2	−457·9	24·1	−29·9	90·8	−111·8
	135	−1·2	−15·5	−23·6	−410·3	−2·0	−26·2	−7·3	−98·5
	180	−9·5	−0·1	−251·7	−8·5	−16·0	−0·1	−60·1	−0·7
	225	−1·7	15·5	−46·1	417·7	−3·0	26·1	−9·9	98·9
	270	13·9	18·1	376·7	493·0	23·2	30·7	89·5	115·2
	315	23·8	10·1	641·9	273·5	40·0	17·2	151·9	63·5

core location; (*e*) the probability of acceptance by the spectrograph of muons deflected in the geomagnetic field; and (*f*) the mean momentum ascribed to events with known deflection in the magnetic field of the spectrograph.

Table 2. As for table 1 but for muons of momentum 15 and 50 GeV/c at sea level

Sea level momentum (GeV/c)		15·0		15·0		50·0		50·0	
Height (km)		5·0		7·5		7·5		15·0	
	ϕ (deg)	Deflection of muons (m)							
		East	North	East	North	East	North	East	North
$\theta = 0°$	0	4·0	0·0	8·9	0·0	2·7	0·0	10·9	0·0
$\theta = 10°$	0	5·8	0·0	13·0	0·0	4·0	0·0	16·1	0·0
	45	5·4	−1·2	12·0	−2·7	3·7	−0·8	14·8	−3·3
	90	4·2	−1·8	9·3	−4·0	2·9	−1·2	11·4	−4·9
	135	2·8	−1·3	6·3	−3·0	1·9	−0·9	7·8	−3·7
	180	2·2	0·0	5·0	0·0	1·5	0·0	6·1	0·0
	225	2·4	2·3	5·8	4·0	0·5	4·1	6·3	6·9
	270	3·6	3·2	8·6	5·4	0·9	5·6	9·4	9·4
	315	5·0	2·9	11·6	4·4	2·4	6·3	13·4	8·9
$\theta = 20°$	0	8·1	0·0	18·1	0·0	5·6	0·0	22·3	0·0
	45	7·3	−2·5	16·2	−5·5	5·0	−1·7	20·0	−6·8
	90	4·8	−3·9	10·7	−8·7	3·3	−2·7	13·2	−10·7
	135	1·7	−3·0	3·9	−6·8	1·2	−2·1	4·8	−8·3
	180	0·3	0·0	0·7	0·0	0·2	0·0	0·9	0·0
	225	0·8	3·5	2·9	7·3	−1·8	3·7	1·7	10·0
	270	3·6	5·2	9·4	10·0	−0·6	6·9	9·2	15·0
	315	6·5	4·4	15·4	7·5	2·5	8·0	17·5	13·1
$\theta = 30°$	0	11·3	0·0	25·1	0·0	7·8	0·0	31·0	0·0
	45	10·1	−4·0	22·4	−8·8	7·0	−2·7	27·7	−10.9
	90	6·1	−6·7	13·6	−14·8	4·2	−4·6	16·8	−18·4
	135	0·6	−5·5	1·4	−12·2	0·4	−3·8	1·7	−15·1
	180	−2·1	−0·0	−4·7	−0·0	−1·5	−0·0	−5·9	−0·0
	225	−0·8	5·4	−0·2	12·2	−4·3	3·6	−3·1	14·9
	270	4·3	7·9	11·8	16·1	−1·5	8·5	11·0	22·3
	315	9·0	6·0	21·3	10·9	3·6	9·4	24·3	17·6
$\theta = 40°$	0	16·1	0·0	35·7	0·1	11·2	0·0	44·4	0·0
	45	14·7	−5·9	32·6	−13·1	10·2	−4·1	40·5	−16·3
	90	8·8	−10·9	19·5	−24·2	6·1	−7·6	24·2	−30·1
	135	−0·8	−9·5	−1·7	−21·2	−0·5	−6·6	−2·1	−26·3
	180	−5·8	−0·0	−12·9	−0·0	−4·0	−0·0	−16·0	−0·0
	225	−2·8	8·9	−3·8	20·5	−6·9	4·5	−8·7	24·2
	270	6·5	12·0	17·1	25·5	−1·3	11·0	16·7	33·7
	315	13·5	8·0	31·4	15·3	6·2	10·7	36·5	23·1

4.3. A check on the validity of the calculations

As an overall check on the validity of the calculations, the variation of the muon charge ratio for EAS recorded in each area of high detection probability in the array at various azimuth angles was predicted for all momenta greater than 1 GeV/c and for a mean height of origin of 5 km, using the appropriate experimental lateral distribution function ratio for the X^+Y^+ area of core impact (see figure 2) it was found to be sensibly the same as for the X^+Y^- area with reversed azimuth angle, so that to improve the statistical accuracy the events in the two regions were added together. Thus the events in the 0°–45° bin of the X^+Y^- area were added to those in the 315°–360° bin of the X^+Y^- area etc. The resulting charge ratio variation is shown in figure 6 and it will be seen that there

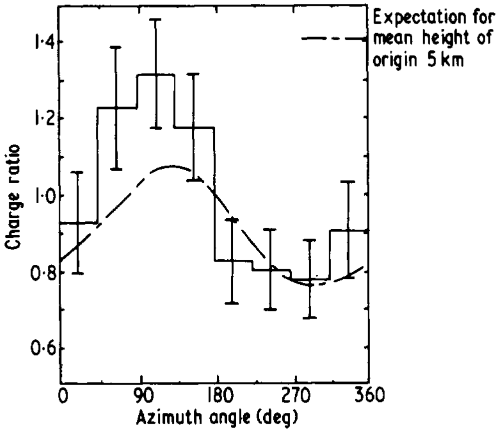

Figure 6. The variation of the charge ratio of muons of momentum 1 GeV/c with EAS azimuth angle for those showers recorded with cores in the region X^+Y^+ and X^+Y^-.

is near agreement between prediction and the observations. In a similar way, since the predicted variation of charge ratio for the $X^-Y^\pm$ area is symmetrical about 180° azimuth, the data for the 0°–45° azimuth bin were added to those for the 315°–360° azimuth bin and so on, and these results are shown in figure 7. Again there is agreement between expectation and observation indicating the overall validity of the deflection calculations.

4.4. The mean heights of origin of muons EAS

Comparisons between the predicted variation of the charge ratio of muons of given momenta with their height of origin and the observed charge ratios has provided an estimation of the heights of origin of muons.

To improve statistics the data were combined as described above and the mean charge ratio expressed as $N(\mu^+)/N(\mu^-)$ was found. Where appropriate the range of azimuth angle over which the mean was taken was limited to include ratios which were either less than or greater than one, but not both. In the case of the $X^-Y^\pm$ area no limitation was imposed, as the predicted charge ratio was greater than unity throughout the range 0°–360°. For the X^+Y^+/X^+Y^- combined areas, the range taken was 180° through 360° to 45°. In this range the predicted ratio was always less than unity.

Thus for all the values of mean momentum bands there are two estimates of the height of origin: one for the $X^-Y^\pm$ area in which the predicted charge ratio is greater than unity and one for the combined X^+Y^+/X^+Y^- area over the azimuth range giving a charge ratio less than unity. Figure 8 shows the variation of these two observed mean charge ratios with sea level momentum. Although the bulk of the data separate into regions of charge ratio greater than and less than 1·0 as expected, there is some indication

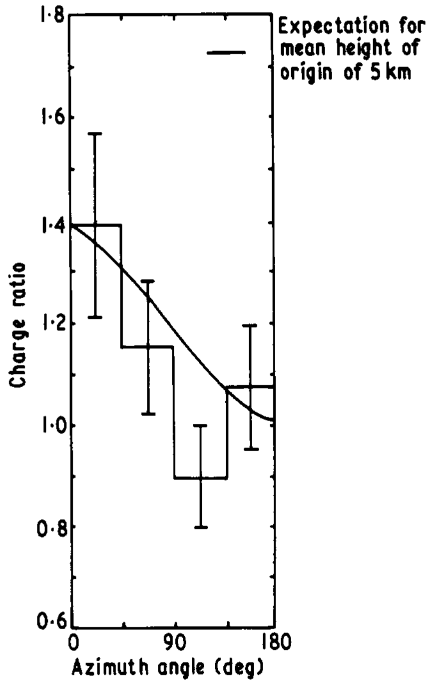

Figure 7. As for figure 6 but for showers recorded with cores in the region $X^- Y^{\pm}$.

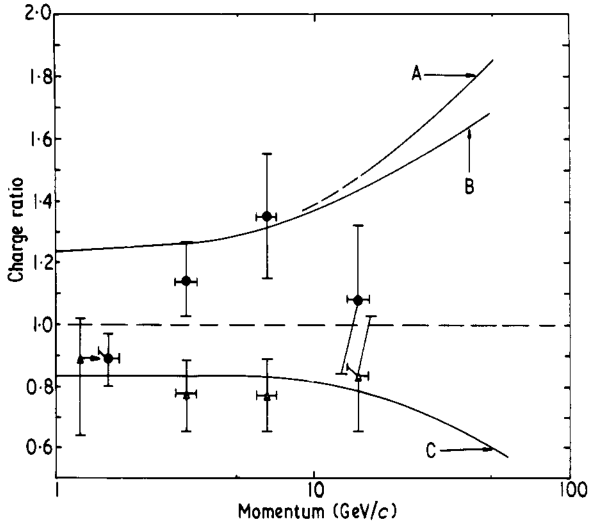

Figure 8. The observed variation of the muon charge ratio for various muon momenta for data grouped to give a charge ratio of greater than unity (-◆-) or less than unity (-+-). Curve A; $A = 1$, $n_s \propto E_p^{1/4}$; curve B: $A = 56$, $n_s \propto E_p^{1/2}$; curve C: $A = 1$, $n_s \propto E_p^{1/4}$ or $A = 56$, $n_s \propto E_p^{1/2}$.

of the breakdown of the effect at low momenta, for which no explanation is presently available.

The estimated heights of origin for muons are shown in figure 9. The vertical error bars indicate the one standard deviation limits due to poissonian errors on the number of events combined with the error due to the uncertainty in the experimental lateral distribution function; the horizontal error bars represent a 10% uncertainty in momentum. The predicted mean heights of origin expected from simulation of showers initiated by primary protons are shown by the full line.

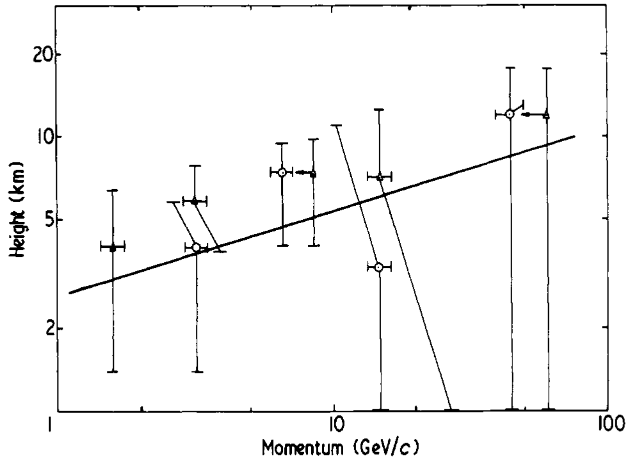

Figure 9. The mean heights of origin of muons derived from the measured charge ratios. The full line represents the data from simulations of proton-initiated showers.

5. Comparison of the heights of origin of muons with data from other experiments

The overall production height of muons greater than 1 GeV/c recorded at a particular core distance was derived from the calculated muon momentum spectrum and the following relation. If the height of origin is denoted by $H(p, r)$ and the momentum spectrum $S(p)\,\mathrm{d}p$, then

$$\langle H(r)\rangle = \frac{\int_1^{30} H(p, r)S(p)\,\mathrm{d}p}{\int_1^{30} S(p)\,\mathrm{d}p}.$$

The value of $\langle H(r)\rangle$ may be regarded as the expectation value for an experiment measuring the height of production of muons of momentum in the range 1–30 GeV/c with no momentum resolution.

In order to compare with the results of earlier experiments which have used a low energy cut-off of about 300 MeV, it is necessary to extend the present results below 1 GeV/c. The use of the above equation is probably accurate for a momentum of 0·5 GeV/c, and this point has been taken as the mean of the momentum band 0 3–1 GeV/c. The relative contribution to the overall spectrum from those muons of momentum 0·3–1 GeV/c was obtained from simulated showers for each distance interval, and a weighted mean height for all muons of momenta greater than 0·3 GeV/c was obtained. The data from the present experiment so adjusted for a low energy cut-off are shown in figure 10.

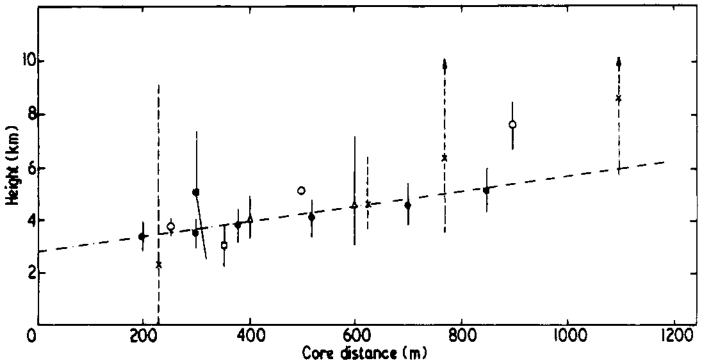

Figure 10. The height of production of muons of momenta greater than 0·3 GeV/c in showers of primary energy of about 10^{17} eV as a function of core distance. The results from other experiments at comparable core distances are also shown. The line shown is the best fit to the present results. ● Trigonometric method, ■ geomagnetic method, present work; ○ Suri (1966); △ Baxter (1967); × Linsley and Scarsi (1962); □ de Beer (1960).

5.1. The results of other measurements

de Beer *et al* (1970) used a method very similar to that described here to predict the angular distribution of muons of momenta greater than 1 GeV/c, and confirmed the general results of their calculations with data from measurements using spark-chamber telescopes (see de Beer *et al* 1962). Figure 10 includes a point computed by the present authors from the data of de Beer (1960).

Suri (1966) measured the mean radius of curvature of the shower front by a fast-timing method, and Baxter (1967) determined the mean height from studies of the Haverah Park array water Čerenkov detector signal profile. Both these measurements were made at the Haverah Park array for muons with a lower cut-off energy of about 0·3 GeV. Their results, shown in figure 10 are in agreement with those from the present experiment. Also shown in figure 10 are the results of Linsley and Scarsi (1962), obtained by timing the delay of each muon with respect to the shower front in the Volcano Ranch array (these latter measurements have been adjusted to allow for the change in observation altitude).

6. The extension of shower simulations to predict the distribution in angles made by muons with the shower core and the distortion in the charge ratio

6.1. A comparison of distributions in $(\psi_p - \psi_0)$ and theoretical predictions to test the validity of shower models

The direct method of determining the production height relies on the accuracy of equation (4), and also on the applicability of a simple analysis. The forms of equations (3) and (4) imply that regions of θ, ϕ, X, Y exist where small changes in any of the given values result in large changes in $1/H$. Furthermore, the data for the mean heights of origin shown in figures 4 and 5 show only a slow change with core distance and momentum. It would therefore be unlikely that such a quantity would be strongly dependent upon the mass of the primary particle assumed or the detail of the model used in the simulation of high energy interactions. Accordingly, it was decided to predict the distribution in the angle $(\psi_p - \psi_0)$ expected at various core distances for different shower models and to determine the parameter of this distribution which was most sensitive to the shower model; for example, the mean or standard deviation of the distribution. It may then be possible to compare the predictions with the observed distributions. Such a method may enable a choice between shower models to be made and also makes possible the correct inclusion of the angular effects of Coulomb scattering and geomagnetic deflection, instead of including them in a 'noise' term, as is necessary when determining directly the mean height of origin by the procedure discussed.

6.2.1. The predictions of a distribution in $(\psi_p - \psi_0)$ from the heights of production of muons. In order to predict accurately the form of the distribution in $(\psi_p - \psi_0)$ it is necessary to account for the physical processes occurring in muon production and propagation to sea level, and experimental effects which modify the expected distribution. The momentum at production is calculated for muons of given momentum recorded at various core distances by assuming a constant rate of energy loss of 2·2 MeV g^{-1} cm^2 and this enables the parent pion momentum and the π–μ decay angle to be calculated. The transverse momentum is a major factor in the angular deviation from the core and the distribution used is that quoted for p–light nucleus collisions by Cocconi *et al* (1961). For muons in every core distance and height interval the value of transverse momentum necessary for the fulfillment of equation (1) is obtained and the probability of this value arising from the CKP distribution may be determined. The geomagnetic deflection of the muon is calculated, converted into an angle, and then combined with the π–μ decay angle to produce correction functions in terms of muon directions at production. The opening angle (ie the angle relative to the core direction) is varied from $-50°$ to $+50°$ in 0·1° steps (the wide range is to allow extreme values of $(\psi_p - \psi_0)$ to be represented) and then checked trigonometrically to ensure that only those values are used which constrain the muon to fall inside the chosen distance band. If so, the probability of observing a muon of momentum p in the chosen distance band arising from the height interval H, $H+\mathrm{d}H$ is determined from the CKP distribution. This process is repeated for all height intervals, and the results are weighted by the predicted muon height distribution to obtain a probability distribution in angular deviation from the core for muons at sea level. This distribution is then projected into the measuring plane of the spectrograph, allowing for the non-uniform acceptance of the array due to areas of high trigger probability shown in figure 2, and finally broadened to represent Coulomb scattering in the atmosphere and in the lead shield of the spectrograph together with the measurement errors, by a gaussian error function of width

$$\sigma_{\mathrm{tot}} = (\sigma_{\mathrm{EAS}}^2 + \sigma_{\mathrm{A}}^2 + \sigma_{\mathrm{Pb}}^2)^{1/2}$$

where σ_{EAS} represents the air shower direction measurement errors, σ_{A} the RMS value of Coulomb scattering in the air, and σ_{Pb} the RMS value of scattering in the lead shielding above the spectrograph. The portion of the distribution of angular deviation between $-16°$ and $+16°$ is normalized (arbitrarily) since these limits were imposed on the experimental data, and the mean deviations of the distribution obtained. The cases for data in regions of $r_\perp^+$ and $r_\perp^-$ were treated separately and a separation of the mean of the two $(\psi_p - \psi_0)$ distributions is obtained. This quantity has been found to reflect most strongly the model used. This treatment predicts the change in separation Δ of the means of the two classes of data, for all values of momentum and core distances; it also shows clearly the asymmetry introduced by the areas of high triggering probability of the EAS array (ie it predicts more events in the $r_\perp^+$ region than the $r_\perp^-$ region of the array and also that average value of the $r_\perp^-$ sample is larger than the average value of the $r_\perp^+$ sample of measured events).

Figure 11 shows the comparison of the predictions for Δ, with the appropriate experimental data for a range of core distance and muon momentum. The shape of the curves varies very little with the overall noise σ_{tot}, because the distributions in $(\psi_p - \psi_0)$ are closely gaussian, and as such the separation between the means changes only slowly with the value of the standard deviation of the gaussian distribution used.

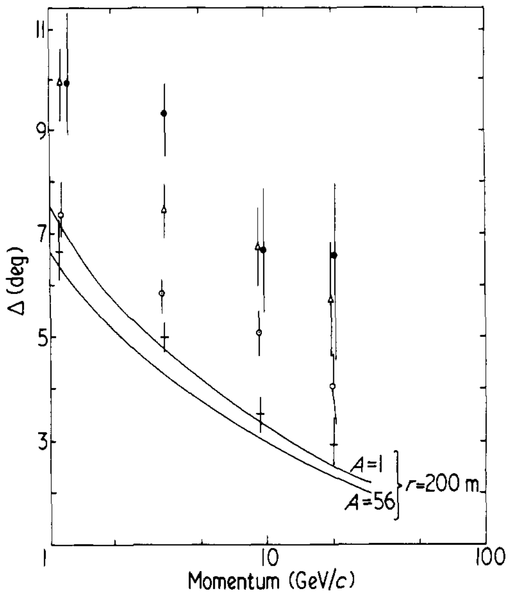

Figure 11. The variation Δ, the separation between the means of the $(\psi_p - \psi_0)$ distribution for data in the $r_\perp^+$ and $r_\perp^-$ regions as a function of momentum, for four core distance intervals. Model predictions for two primary masses are also shown for showers initiated by primary particles of energy 2×10^{17} eV for a core distance of 200 m. + 200 m; ○ 300 m; △ 380 m; ● 520 m.

6.2. Comparison of the observed muon charge ratio with that predicted for various shower models

Models of the development of the muon component of EAS have been compared with observation by using the predicted distribution in height of origin of muons of a particular momentum to derive the charge distortion expected due to the earth's magnetic field. In this technique the following weighting procedure was adopted.

If w_i is the predicted fraction of all muons of momentum p at sea level originating from the ith height interval, and r_i the predicted geomagnetic charge ratio of these muons at the point of observation, then the fraction P_i of positive muons arriving at the point of observation from the ith height interval, is

$$P_i = \frac{r_i w_i}{1 + r_i}.$$

The total predicted charge ratio R defined as the ratio of the total number of positive and the total number of negative muons arriving at the point of observation and originating from all height intervals up to the highest is given by

$$R = \sum_{i=0}^{i=i_{\max}} \frac{P_i}{w_i - P_i}.$$

The heights of origin of muons are influenced by the multiplicity of produced particles in interactions and the mass number of the primary particle. A proton-initiated shower with a multiplicity law for the pions produced varying as the quarter power of the primary energy, and the shower initiated by an iron nucleus and having a multiplicity law following the half power law in energy represent two extremes in so far as the mean muon height of origin distributions are concerned. For this reason they have been used to test the sensitivity of expected charge ratio of muons to the distribution in height of origin of muons. The models used were those of Orford and Turver (1970), and the charge ratios were predicted for the mean momentum appropriate to the experimental data. It is interesting to note that the lateral distribution for the shower initiated by an iron nucleus and having a multiplicity law proportional to the half power of the energy is flatter than that for the shower initiated by a primary proton and having the multiplicity law proportional to the quarter power law of energy; therefore, although the heavy particle cascade develops earlier and is more influenced by the geomagnetic field, the effect of the increased deflection on the charge ratio is not so great as it would have been, had the lateral distribution not also flattened. The predicted charge ratios are shown in figure 8 and it is apparent that the ratios, for all momenta except the largest, are the same for the two models.

It may be concluded that the distortion of the charge ratio by the geomagnetic field is not very sensitive to the particular model of shower development used, due to the general compensating feature that all models that have an early development although providing muons with a longer path length in the geomagnetic field also give rise to a flatter lateral distribution at sea level.

7. Conclusion

The mean heights of origin $\langle H \rangle$ of muons of momentum p GeV/c recorded at core distances of r km at sea level in showers produced by primary particles at an energy of about 10^{17} eV have been derived from consideration of the angular deviation of muons from the shower core and the interaction between muons and the geomagnetic field. Data from both methods are consistent with heights of origin specified by the relationship (H in km):

$$H(p, r) = H_0 + \alpha \lg p + r/\beta \text{ (km)}$$

where

$$H_0 = 1{\cdot}68 \pm 0{\cdot}15 \text{ km}$$

$$\alpha = 1{\cdot}74 \pm 0{\cdot}20 \text{ km}$$

and

$$\beta = 0{\cdot}263 \pm 0{\cdot}033$$

for momenta of 1–30 GeV/c and core distances 150–600 m.

The lateral deflection of muons arising from their interaction with the geomagnetic field has been shown in tables 1 and 2 to be, in certain cases, very large. For example a positive muon of momentum 1 GeV/c originating from a height of 5 km in a shower arriving from due north at a zenith angle of 40° at the location of Haverah Park (ie in a direction nearly perpendicular to the geomagnetic lines of force) will be deflected by up to 100 m in an easterly direction.

The effects of these large deflections of muons on the lateral distribution functions for all muons in showers arriving within 40° of the zenith is small however due to the compensating effects of the opposite lateral deflections of positive and negative charged muons. It is noted that, as pointed out by Hillas *et al* (1970), this is not so for showers incident at large zenith angles (greater than 70°) when in certain circumstances the positive and negative muon components of the shower may separate totally, producing a marked effect on the lateral distribution function.

It is shown that both measurements discussed here are in agreement with the expectation for a wide range of simulated showers and neither of the measurements appears very sensitive to the average value of the mass number of the primary particle or the rapidity of the cascade development.

Acknowledgments

We are indebted to Professor G D Rochester FRS and Professor A W Wolfendale for their interest and encouragement of this work, and to colleagues in the University of Leeds for the provision of full data on the air showers.

The Science Research Council is thanked for the award of a Research Grant and a Research Studentship to ACM.

References

Baxter A J 1967 *PhD Thesis* University of Leeds
de Beer J F 1960 *PhD Thesis* University of Potchefstroom
de Beer J F, Cranshaw T E and Parham A G 1962 *Phil. Mag.* 7 499–514
de Beer J F, de Villiers E J, Reinecke J P L and Venter F A 1970 *Acta Phys. Hung.* **29** 657–60
Clark G, Earl J, Kraushaar W, Linsley J, Rossi B and Sherb F 1958 *Suppl. Nuovo. Cim.* **8** 628–52
Cocconi G, Koester L J and Perkins D H 1961 *Lawrence Radiation Laboratory, High Energy Physics Study Seminars* vol 28 No 2 UCID-1444, 1
Earnshaw J C 1968 *PhD Thesis* University of Durham
Earnshaw J C, Maslin G C and Turver K E 1968 *Can. J. Phys.* **46** S115–8
Earnshaw J C, Orford K J, Rochester G D, Somogyi A J and Turver K E 1967 *Proc. Phys. Soc.* **90** 91–108
Hillas A M, Hollows J D, Hunter H W and Marsden D J 1970 *Acta Phys. Hung.* **29** 533–8
Hollows J F 1968 *PhD Thesis* University of Leeds
Kamiya Y, Sugisaka S, Ueno H, Kato S, and Sekido Y 1962 *J. Phys. Soc. Japan* **17** A3 315–8
Khristiansen G B 1958 *Zh. eksp. teor. Fiz.* **34** 956–61
Linsley J and Scarsi L 1962 *Phys. Rev. Lett.* **9** 123–5
Oren Y 1959 *Bull. Res. Coun. Israel* **8** 103–12
Orford K J, Turver K E and Walton A B 1968 *Can. J. Phys.* **46** S119–22
Orford K J and Turver K E 1970 *Acta Phys. Hung.* **29** 585–92
Somogyi A J 1966 *Ann. Phys., Lpz* **17** 221–32
Suri A 1966 *PhD Thesis* University of Leeds
Tennent R M 1967 *Proc. Phys. Soc.* **92** 622–31